



# Curso – Resistencia de materiales [15153]

## Clase 8 – Flexión y esfuerzo en vigas II

Plan de estudios - Ingeniería Civil en Mecánica

Profesores: Matías Pacheco Alarcón ([matias.pacheco@usach.cl](mailto:matias.pacheco@usach.cl))

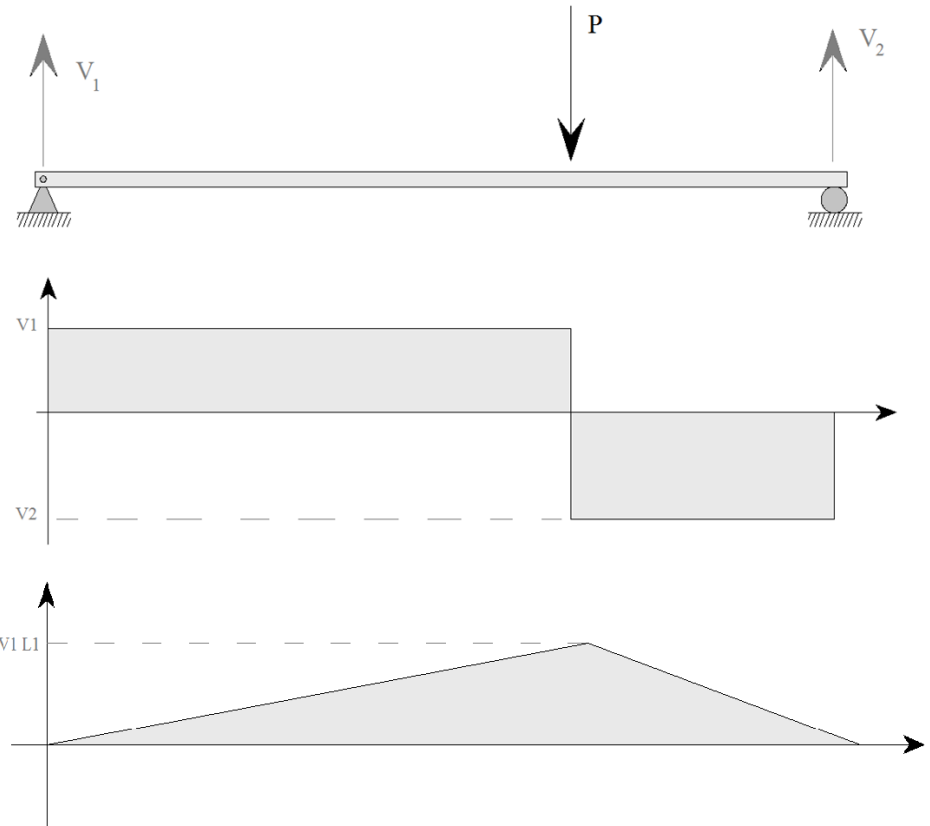
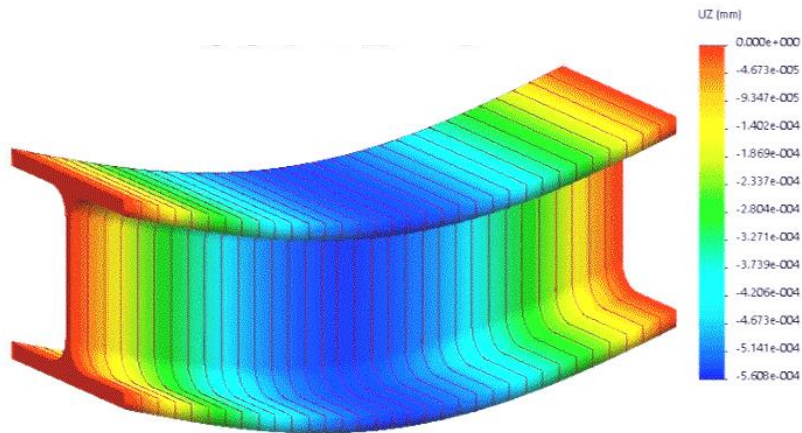
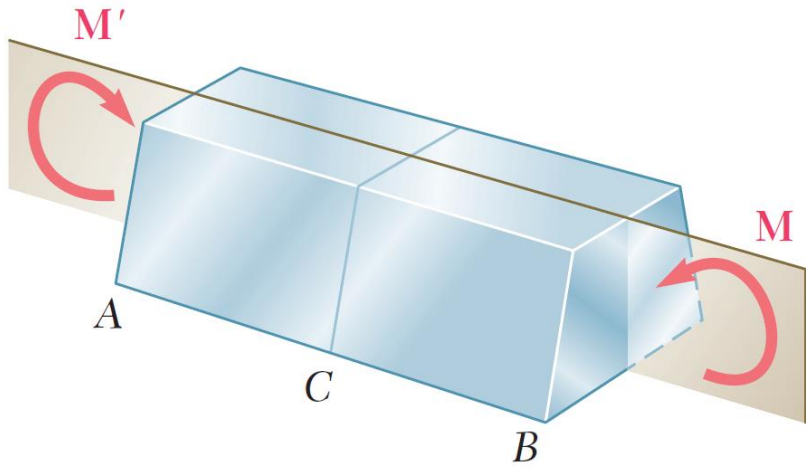
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Santiago de Chile, Mayo 2019



# Resumen clase anterior





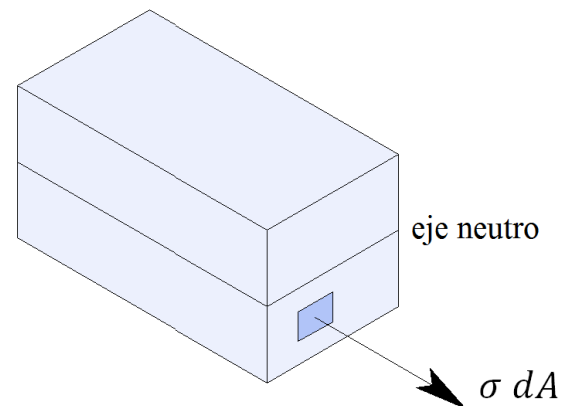
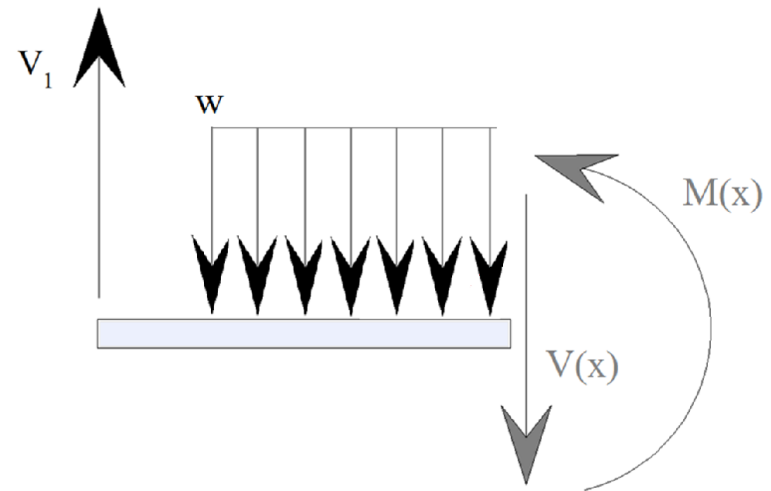
## Resumen clase anterior

$$\sum F_{axial} :$$

$$\bar{y} = 0$$

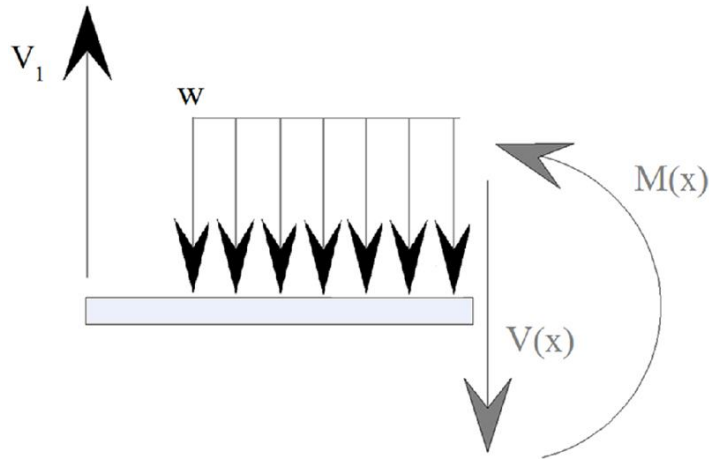
$$\sum M_{flector} :$$

$$\sigma = \frac{M y}{I_{EN}}$$

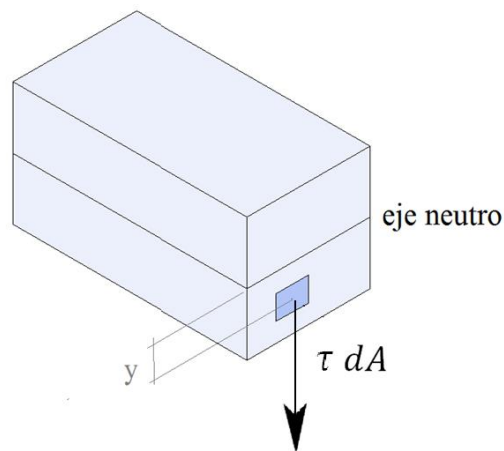




## Resumen clase anterior



$$\int \tau dA = V \quad ???$$

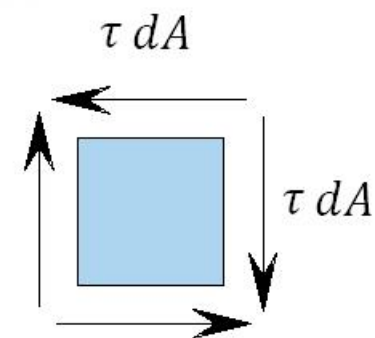
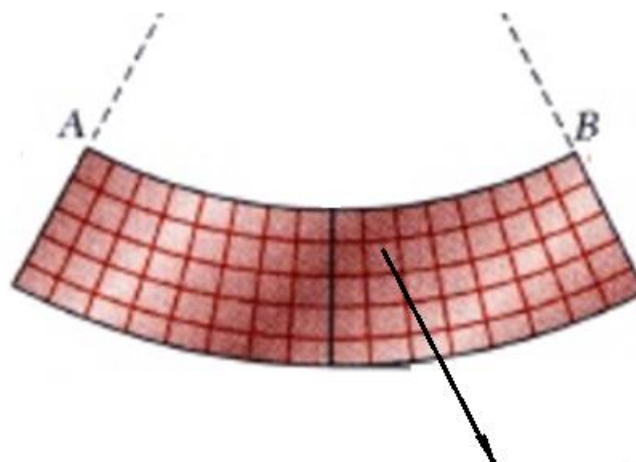
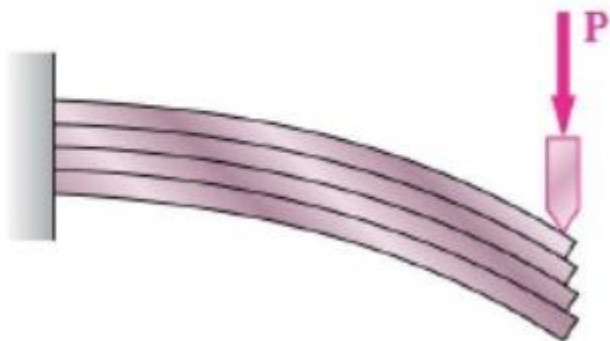
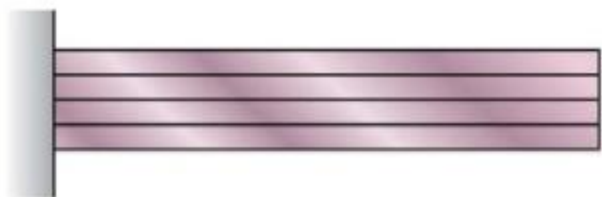


- La distribución de esfuerzos es desconocida.



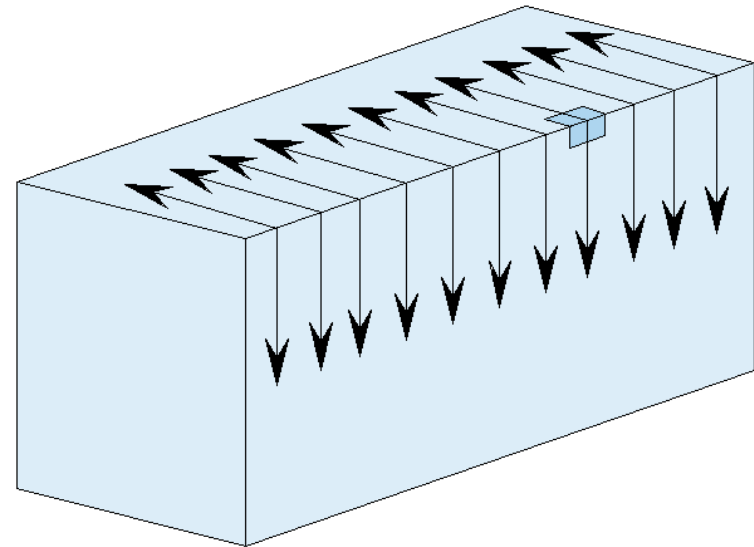
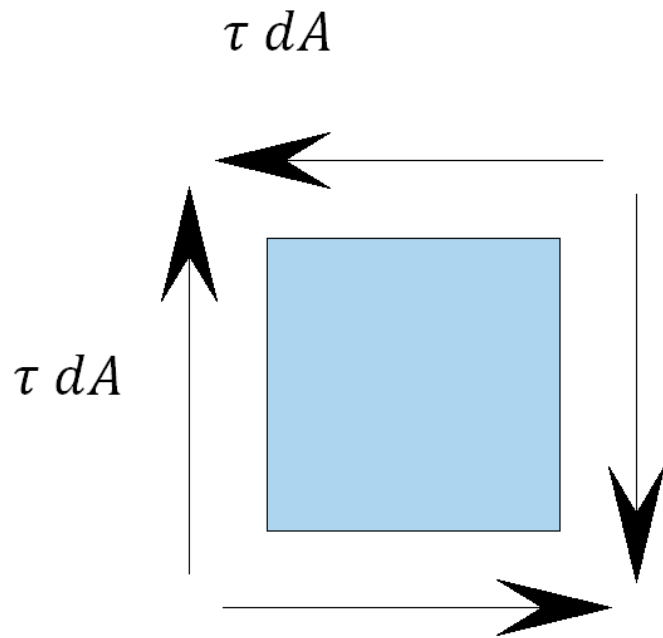
# Esfuerzo de corte en flexión

Considere los siguientes casos



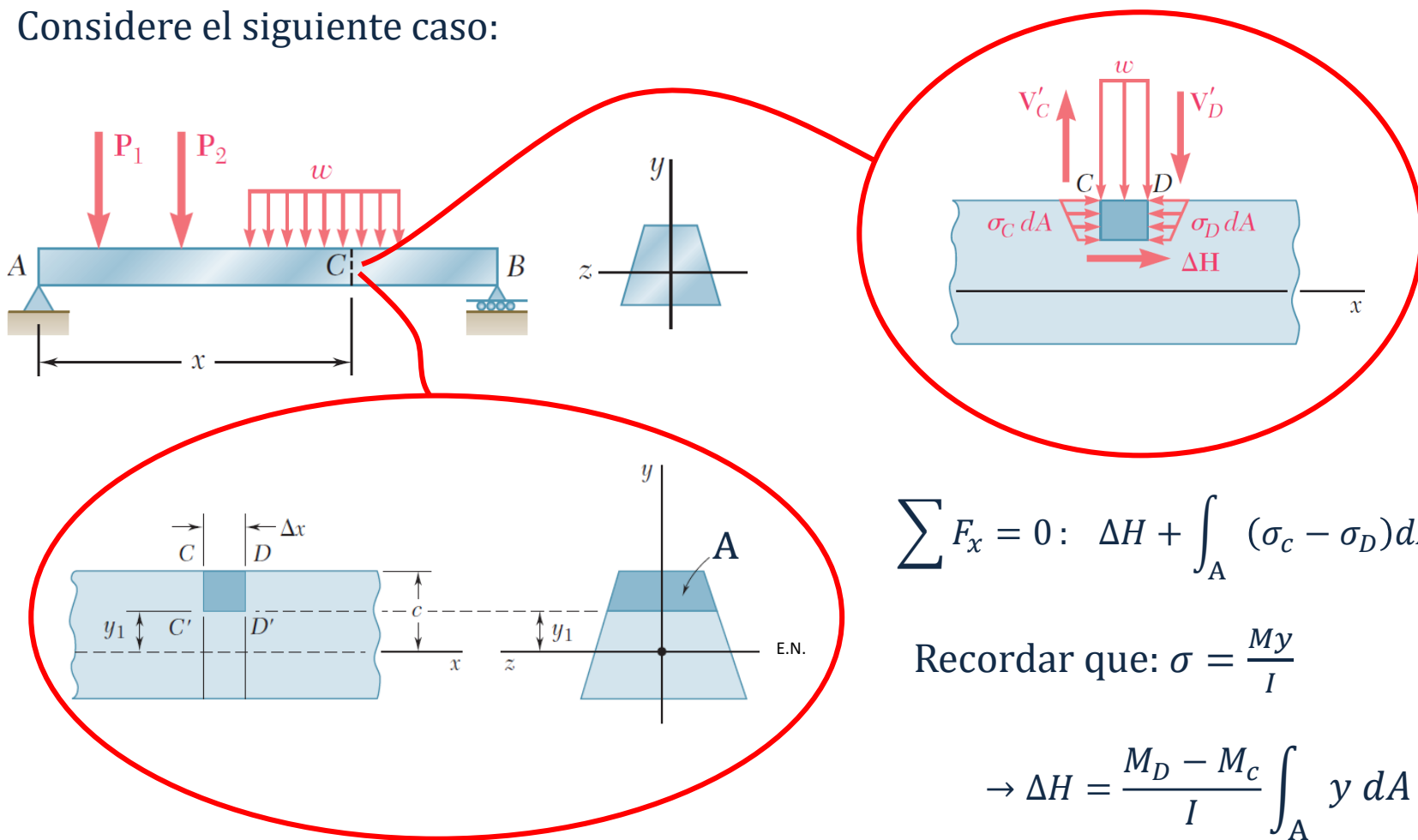


## Esfuerzo de corte en flexión



# Esfuerzo de corte en flexión

Considere el siguiente caso:



$$\sum F_x = 0: \Delta H + \int_A (\sigma_c - \sigma_D) dA = 0$$

Recordar que:  $\sigma = \frac{My}{I}$

$$\rightarrow \Delta H = \frac{M_D - M_C}{I} \int_A y dA$$



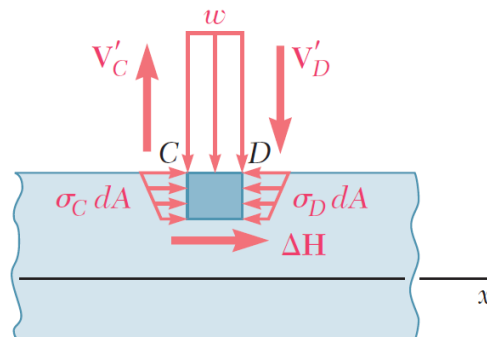
## Esfuerzo de corte en flexión

Primer momento de área respecto al eje neutro de la porción A:  $Q = \int_A y dA$

$$\Delta H = \frac{M_D - M_C}{I} \int_A y dA = \frac{M_D - M_C}{I} Q = \frac{\Delta M}{I} Q = \frac{(dM/dx)\Delta x}{I} Q = \frac{VQ}{I} \Delta x$$

Luego, el esfuerzo cortante horizontal por unidad de longitud o flujo de corte es:

$$q = \frac{\Delta H}{\Delta x} = \frac{VQ}{I}$$

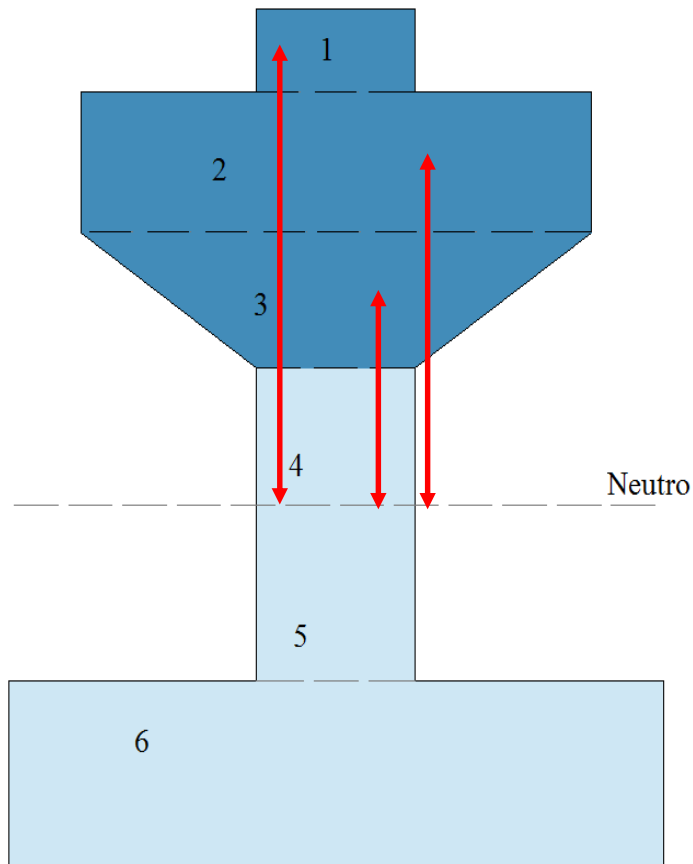






## Primero momento de área

Si centroides y áreas 1, 2 y 3 son conocidos:



$$\int_{A_3}^{A_1} y dA = \bar{y}A_{31} = Q$$

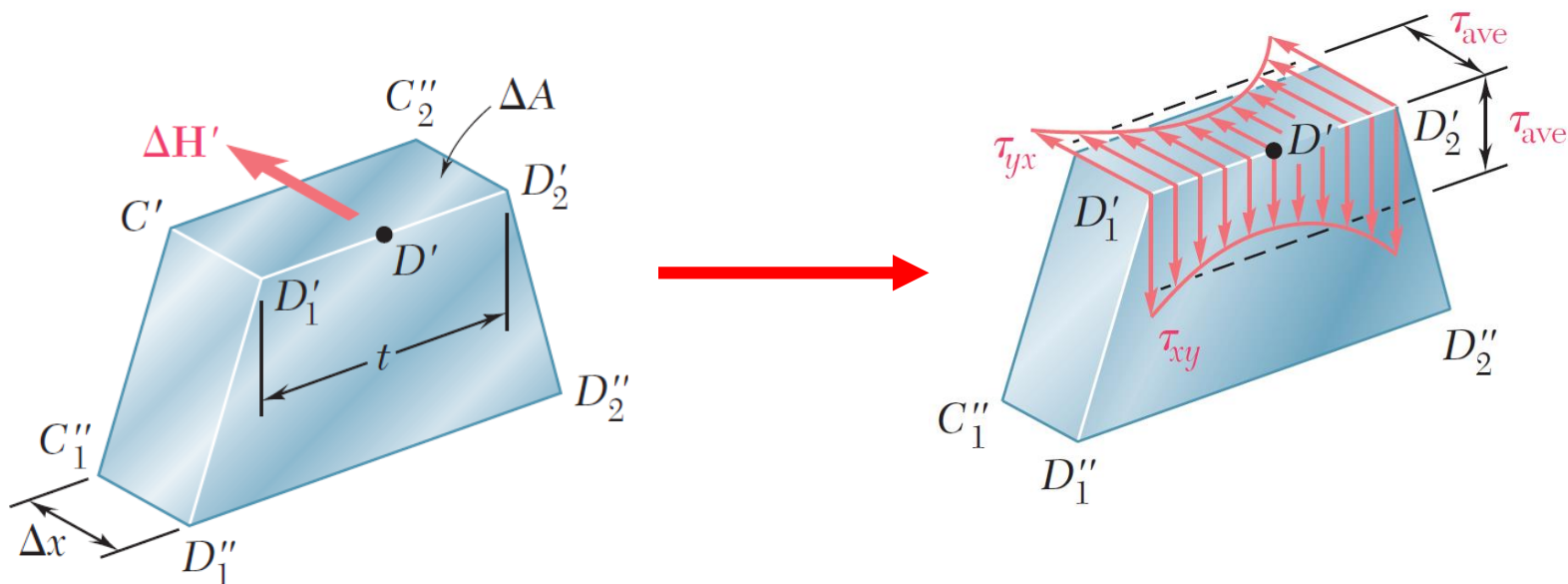
$$\int_{y_3}^{y_1} yt(y) dy = Q$$

$$\int_{y_3}^{y_2} yt(y) dy + \int_{y_2}^{y_1} yt(y) dy + \int_{y_1}^{y_{max}} yt(y) dy = Q$$

$$\bar{y}_3A_3 + \bar{y}_2A_2 + \bar{y}_1A_1 = Q$$

## Esfuerzo de corte en flexión

La distribución es casi constante en el ancho



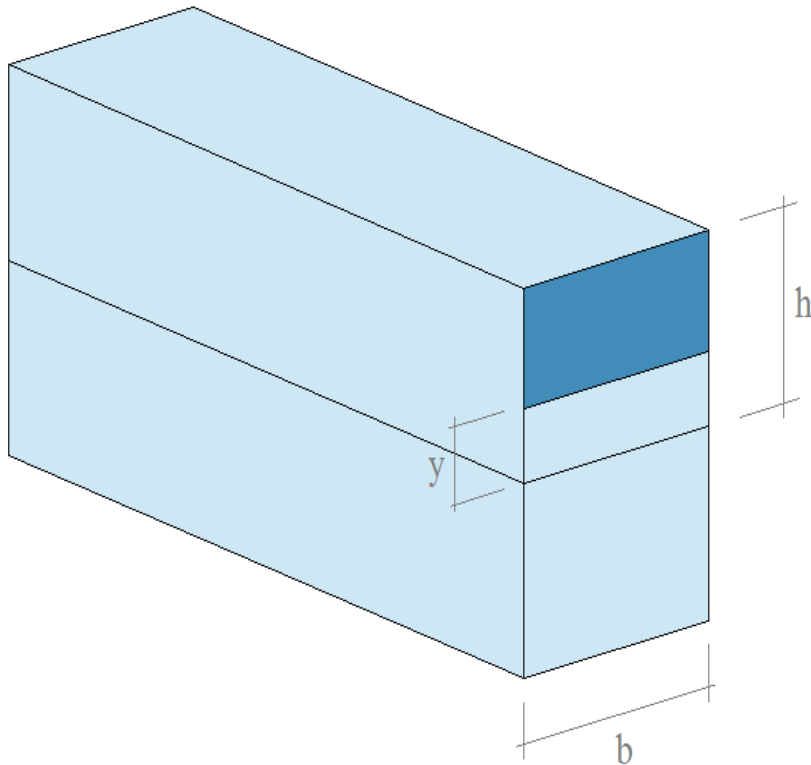
$$\tau_{avg} = \frac{\Delta H}{\Delta A} = \frac{VQ}{I} \frac{\Delta x}{t\Delta x} \rightarrow \tau_{avg} = \frac{VQ}{It}$$

Formula de Collignon-Jourawsky



# Esfuerzo de corte en flexión

Sección rectangular



$$\tau = \frac{VQ}{It}; \quad Q = \frac{(h+y)}{2} (h-y)b$$

$$\tau = V \frac{(h^2 - y^2)}{2I}$$

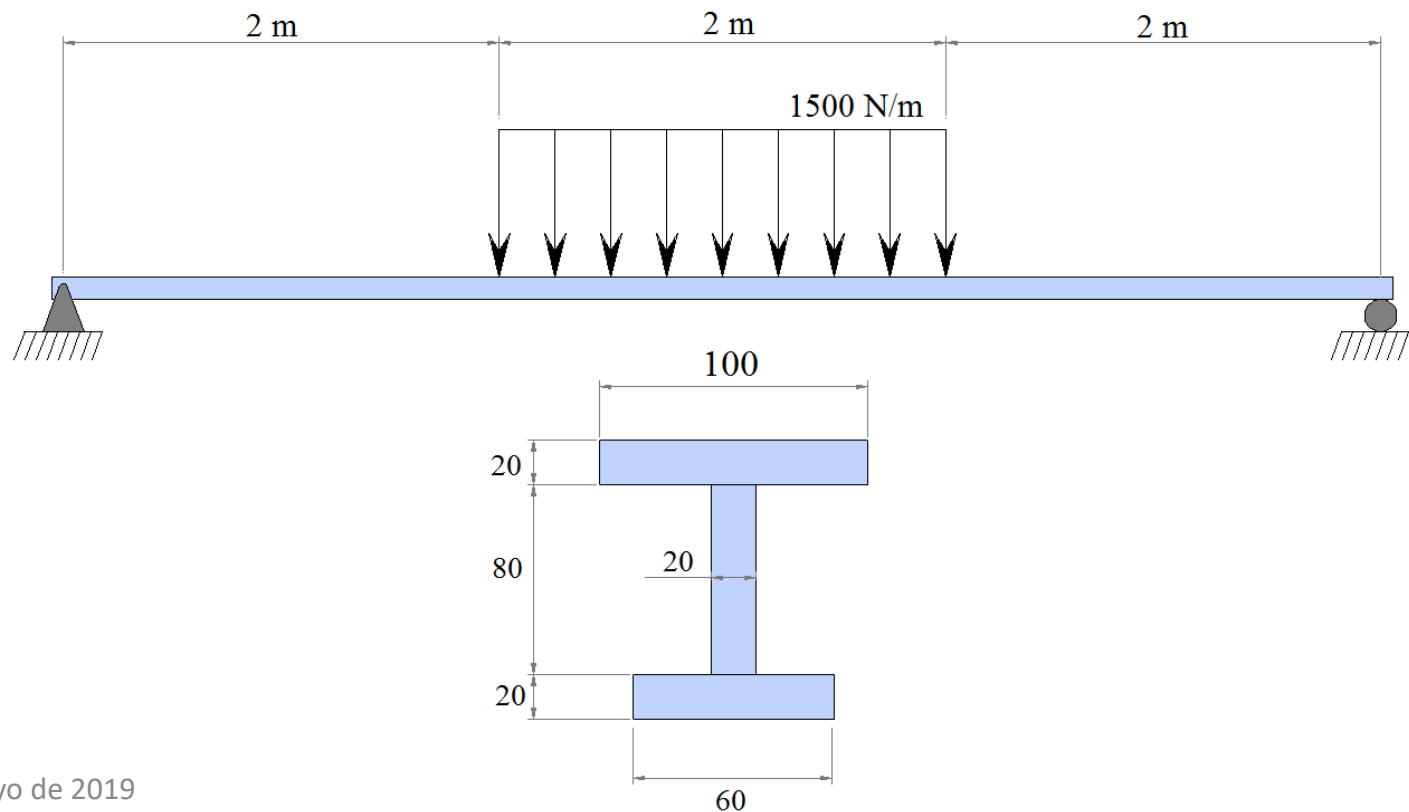
$$\tau = \frac{3}{4} V \frac{(h^2 - y^2)}{bh^3}$$



## Ejemplo:

Se tiene la viga de la figura cargada con una fuerza distribuida en su centro. El detalle de la sección transversal se muestra en la figura. Determinar:

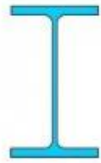
1. Las zonas con mayor fuerza de corte y con mayor momento flector
2. Los esfuerzos de corte máximo en las placas unidas
3. Máximo esfuerzo normal.





## Perfiles comerciales en vigas

- Norma europea (DIN)



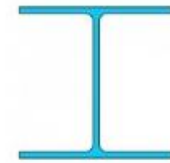
IPE



IPN



HE



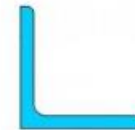
HL



UPE



UPN

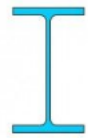


L

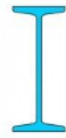


## Perfiles comerciales en vigas

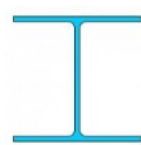
- Norma Americana (ASTM):



W



S



HP



C



MC



L

- Norma Inglesa:



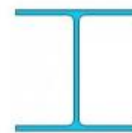
UB



J



UC



UBP



PCF



CH



# Perfiles comerciales en vigas

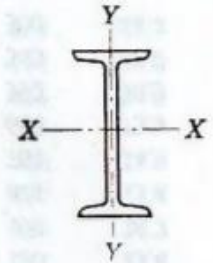


TABLA B-2. Perfiles I (vigas de ala ancha), americanos (W) (unidades SI)

Denominación	Masa (aprox.) (kg/m)	Área transversal (mm <sup>2</sup> )	Altura (mm)	Ala (o patin)			Eje X-X			Eje Y-Y		
				Ancho (mm)	Espesor (mm)	Espesor de alma (mm)	$I$ (10 <sup>6</sup> mm <sup>4</sup> )	$S = \frac{I}{c}$ (10 <sup>3</sup> mm <sup>3</sup> )	$r = \frac{I}{\sqrt{I/A}}$ (mm)	$I$ (10 <sup>6</sup> mm <sup>4</sup> )	$S = \frac{I}{c}$ (10 <sup>3</sup> mm <sup>3</sup> )	$r = \frac{I}{\sqrt{I/A}}$ (mm)
<b>W920 × 446</b>	447.2	57 000	933	423	42.7	24.0	8 470	18 200	386	540	2 550	97.3
× 417	418.1	53 300	928	422	39.9	22.5	7 880	17 000	385	501	2 370	97.0
× 387	387.0	49 300	921	420	36.6	21.3	7 180	15 600	382	453	2 160	95.8
× 365	364.6	46 400	916	419	34.3	20.3	6 710	14 600	380	421	2 010	95.2
× 342	342.4	43 600	912	418	32.0	19.3	6 250	13 700	379	390	1 870	94.6
× 313	312.7	39 800	932	309	34.5	21.1	5 480	11 800	371	170	1 100	65.4
× 289	288.6	36 800	927	308	32.0	19.4	5 040	10 900	370	156	1 020	65.2
× 271	271.7	34 600	923	307	30.0	18.4	4 720	10 200	369	145	946	64.8
× 253	253.7	32 300	919	306	27.9	17.3	4 370	9 520	368	134	874	64.3
× 238	238.3	30 400	915	305	25.9	16.5	4 060	8 880	366	123	806	63.6
× 223	224.2	28 600	911	304	23.9	15.9	3 770	8 270	363	112	738	62.7
× 201	201.3	25 600	903	304	20.1	15.2	3 250	7 200	356	94.4	621	60.7
<b>W840 × 359</b>	359.4	45 800	868	403	35.6	21.1	5 910	13 600	359	389	1 930	92.2
× 329	329.4	42 000	862	401	32.4	19.7	5 350	12 400	357	349	1 740	91.2
× 299	299.3	38 100	855	400	29.2	18.2	4 790	11 200	355	312	1 560	90.4
× 226	226.6	28 900	851	294	26.8	16.1	3 400	7 990	343	114	774	62.8
× 210	210.8	26 800	846	293	24.4	15.4	3 110	7 340	340	103	700	61.8
× 193	193.5	24 700	840	292	21.7	14.7	2 780	6 630	336	90.3	618	60.5
× 176	176.0	22 400	835	292	18.8	14.0	2 460	5 900	331	78.2	536	59.1





# Perfiles comerciales en vigas

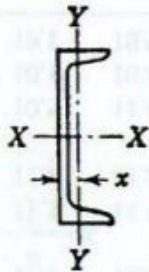


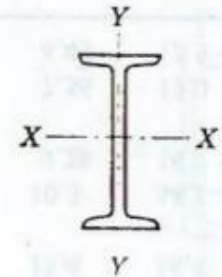
TABLA B-4. Perfiles C (canales), americanos (unidades SI)

Denominación	Masa (aprox.) (kg/m)	Área transversal (mm <sup>2</sup> )	Altura (mm)	Ala (o patin)			Eje X-X			Eje Y-Y			
				Ancho (mm)	Espesor (mm)	Espesor de alma (mm)	$I$ (10 <sup>6</sup> mm <sup>4</sup> )	$S = \frac{I}{c}$ (10 <sup>3</sup> mm <sup>3</sup> )	$r = \frac{I}{A}$ (mm)	$I$ (10 <sup>6</sup> mm <sup>4</sup> )	$S = \frac{I}{c}$ (10 <sup>3</sup> mm <sup>3</sup> )	$r = \frac{I}{A}$ (mm)	$x$ (mm)
C380 × 74	74.4	9 480	381	94	16.5	18.2	168	881	133	4.60	62.4	22.0	20.3
× 60	59.4	7 570	381	89	16.5	13.2	145	760	138	3.84	55.5	22.5	19.7
× 50	50.5	6 430	381	86	16.5	10.2	131	687	143	3.39	51.4	23.0	20.0
C310 × 45	44.7	5 690	305	80	12.7	13.0	67.3	442	109	2.12	33.6	19.3	17.0
× 37	37.1	4 720	305	77	12.7	9.8	59.9	393	113	1.85	30.9	19.8	17.1
× 31	30.8	3 920	305	74	12.7	7.2	53.5	351	117	1.59	28.2	20.1	17.5
C250 × 45	44.5	5 670	254	76	11.1	17.1	42.8	337	86.9	1.60	26.8	16.8	16.3
× 37	37.3	4 750	254	73	11.1	13.4	37.9	299	89.4	1.40	24.3	17.1	15.7
× 30	29.6	3 780	254	69	11.1	9.6	32.7	257	93.0	1.16	21.5	17.5	15.3
× 23	22.6	2 880	254	65	11.1	6.1	27.8	219	98.2	0.922	18.8	17.9	15.9
C230 × 30	29.8	3 800	229	67	10.5	11.4	25.5	222	81.9	1.01	19.3	16.3	14.8
× 22	22.3	2 840	229	63	10.5	7.2	21.3	186	86.6	0.806	16.8	16.8	14.9
× 20	19.8	2 530	229	61	10.5	5.9	19.8	173	88.6	0.716	15.6	16.8	15.1





# Perfiles comerciales en vigas

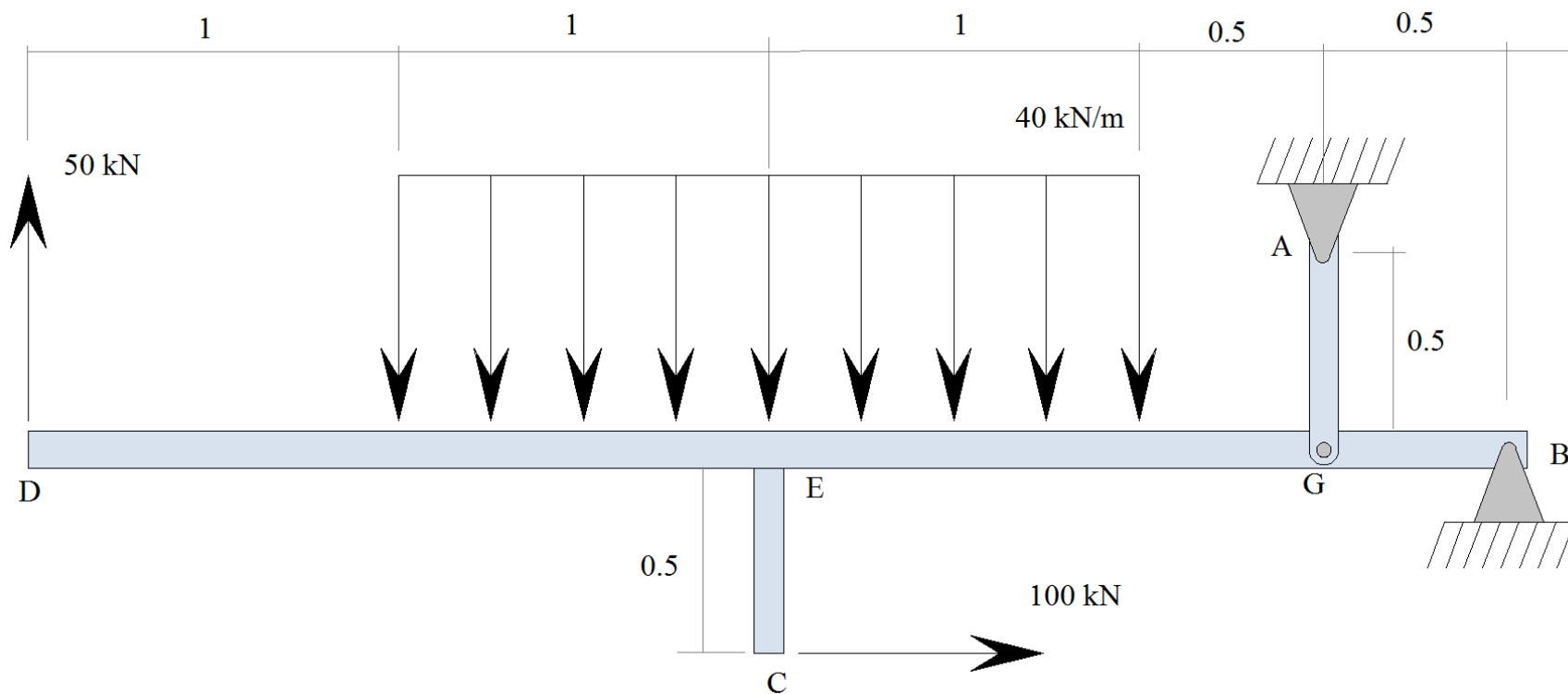


**TABLA B-3. Perfiles I (vigas normales), americanos (S) (unidades SI)**

Denominación	Masa (aprox.) (kg/m)	Área transversal (mm <sup>2</sup> )	Altura (mm)	Ala (o patín)			Eje X-X			Eje Y-Y		
				Ancho (mm)	Espesor (mm)	Espesor de alma (mm)	$I$ (10 <sup>6</sup> mm <sup>4</sup> )	$S = \frac{I}{c}$ (10 <sup>3</sup> mm <sup>3</sup> )	$r = \frac{I}{A}$ (mm)	$I$ (10 <sup>6</sup> mm <sup>4</sup> )	$S = \frac{I}{c}$ (10 <sup>3</sup> mm <sup>3</sup> )	$r = \frac{I}{A}$ (mm)
S610 × 179	178.9	22 800	610	204	28.0	20.3	1 260	4 140	235	35.1	345	39.3
× 157.6	158.0	20 100	610	200	28.0	15.9	1 180	3 870	242	32.8	328	40.4
× 149	149.2	19 000	610	184	22.1	19.0	997	3 270	229	20.1	218	32.5
× 134	133.9	17 100	610	181	22.1	15.8	938	3 070	234	18.9	209	33.3
× 118.9	119.1	15 200	610	178	22.1	12.7	879	2 880	241	17.9	201	34.3
S510 × 141	141.8	18 100	508	183	23.3	20.3	674	2 660	193	21.0	230	34.1
× 127	126.9	16 200	508	179	23.3	16.6	633	2 490	198	19.5	218	34.7
× 112	111.9	14 300	508	162	20.1	16.3	532	2 100	193	12.5	154	29.6
× 97.3	97.8	12 500	508	159	20.1	12.7	494	1 950	199	11.7	147	30.6
S460 × 104	104.7	13 300	457	159	17.6	18.1	387	1 690	170	10.3	129	27.7
× 81.4	81.6	10 400	457	152	17.6	11.7	335	1 470	180	8.77	115	29.1
S380 × 74	74.6	9 500	381	143	15.8	14.0	203	1 060	146	6.60	92.3	26.4
× 64	63.9	8 150	381	140	15.8	10.4	187	980	151	6.11	87.3	27.4



**Ejemplo:** Se tiene el siguiente sistema de vigas donde los elementos DB y EC son vigas de acero A36 ( $\sigma_y = 240\text{MPa}$ ) y el elemento AG es una barra. Las dimensiones se muestran en metros. Determine el perfil comercial W americano para que el sistema no falle.





Denominación	Masa (aprox.) (kg/m)	Área transversal (mm <sup>2</sup> )	Altura (mm)	Ala (o patin)			Eje X-X			Eje Y-Y		
				Ancho (mm)	Espesor (mm)	Espesor de alma (mm)	$I$ (10 <sup>6</sup> mm <sup>4</sup> )	$S = \frac{I}{c}$ (10 <sup>3</sup> mm <sup>3</sup> )	$r = \frac{I}{A}$ (mm)	$I$ (10 <sup>6</sup> mm <sup>4</sup> )	$S = \frac{I}{c}$ (10 <sup>3</sup> mm <sup>3</sup> )	$r = \frac{I}{A}$ (mm)
W310 × 52	52.3	6 670	317	167	13.2	7.6	118	747	133	10.3	123	39.2
× 45	44.6	5 690	313	166	11.2	6.6	99.2	634	132	8.55	103	38.8
× 39	38.7	4 940	310	165	9.7	5.8	85.1	549	131	7.27	88.1	38.4
× 35	32.8	4 180	313	162	10.8	6.8	85.8	413	125	1.92	37.8	21.4
× 28	28.4	3 610	309	102	8.9	6.0	54.3	351	123	1.58	31.0	20.9
× 24	23.8	3 040	305	101	6.7	5.6	42.7	280	119	1.16	22.9	19.5
× 21	21.1	2 690	303	101	5.7	5.1	37.0	244	117	0.983	19.5	19.1
W250 × 167	167.4	21 300	289	265	31.8	19.2	300	2 080	119	98.8	746	68.1
× 149	148.9	19 000	282	263	28.4	17.3	259	1 840	117	86.2	656	67.4
× 131	131.1	16 700	275	261	25.1	15.4	221	1 610	115	74.5	571	66.8
× 115	114.8	14 600	269	259	22.1	13.5	189	1 410	114	64.1	495	66.2
× 101	101.2	12 900	264	257	19.6	11.9	164	1 240	113	55.5	432	65.6
× 89	89.6	11 400	260	256	17.3	10.7	143	1 100	112	48.4	378	65.1
× 80	80.1	10 200	256	255	15.6	9.4	126	982	111	43.1	338	65.0
× 73	72.9	9 280	253	254	14.2	8.6	113	891	110	38.8	306	64.6
× 67	67.1	8 550	257	204	15.7	8.9	104	806	110	22.2	218	51.0
× 58	58.2	7 420	252	203	13.5	8.0	87.3	693	108	18.8	186	50.4
× 49	49.0	6 250	247	202	11.0	7.4	70.6	572	106	15.1	150	49.2
× 45	44.9	5 720	266	148	13.0	7.6	71.1	534	111	7.03	95.1	35.1
× 39	38.7	4 920	262	147	11.2	6.6	60.1	459	110	5.94	80.8	34.7
× 33	32.7	4 170	258	146	9.1	6.1	48.9	379	108	4.73	64.7	33.7
× 28	28.5	3 630	260	102	10.0	6.4	40.0	307	105	1.78	34.8	22.1
× 25	25.3	3 230	257	102	8.4	6.1	34.2	266	103	1.49	29.2	21.5
× 22	22.4	2 850	254	102	6.9	5.8	28.9	227	101	1.23	24.0	20.7
× 18	17.9	2 270	251	101	5.3	4.8	22.4	179	99.3	0.913	18.1	20.0



# ¿Consultas?

## **Curso – Resistencia de Materiales [15153]**

Plan de estudios - Ingeniería Civil en Mecánica

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